

Forces and moments acting on a vehicle, tire-road contact forces

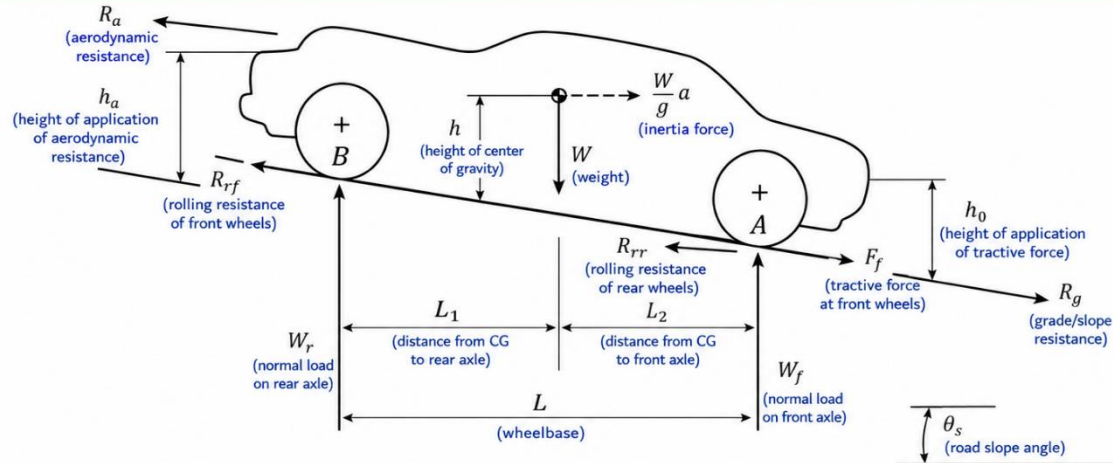
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Sustainable Transportation within the Framework of Green Deal - SPHERE

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Forces acting on a car



LEGEND

R_a - aerodynamic resistance (drag force) (N)
 h_a - height of application of aerodynamic resistance (m)
 R_{rf} - rolling resistance of front wheels (N)
 W_r - normal load on rear axle (N)
 B - rear axle (point B)
 L_1 - distance from center of gravity (CG) to rear axle (m)
 L - wheelbase (m)

h - height of center of gravity (m)
 W - weight of the vehicle (N)
 $\frac{W}{g} a$ - inertia force (N)
 L_2 - distance from center of gravity (CG) to front axle (m)
 W_f - normal load on front axle (N)
 A - front axle (point A)

R_{rr} - rolling resistance of rear wheels (N)
 F_f - tractive (driving) force at front wheels (N)
 h_0 - height of application of tractive force (m)
 R_g - grade (slope) resistance (N)
 θ_s - road slope angle (rad or deg)

EQUATION OF MOTION AND MAXIMUM TRACTIVE EFFORT

$$m \frac{d^2x}{dt^2} = \frac{W}{g} a = F_f + F_r - R_a - R_{rf} - R_{rr} - R_d - R_g \quad \text{----- (1)}$$

By applying **Newton's second law (inertia force)** in the direction of motion, the above equation can be rearranged as

$$F_f + F_r - \left(R_a + R_{rf} + R_{rr} + R_d + R_g + \frac{aW}{g} \right) = 0 \quad \text{----- (2)}$$

or

$$F = R_a + R_r + R_d + R_g + \frac{aW}{g}$$

where

m	– mass of the body (kg)	F_f	– tractive (driving) force (N)
x	– distance (m)	F_r	– grade (slope) resistance force (N)
t	– time (s)	R_a	– aerodynamic resistance (drag force) (N)
W	– weight of the body (N)	R_{rf}	– rolling resistance (front wheels) (N)
g	– acceleration due to gravity (m/s^2)	R_{rr}	– rolling resistance (rear wheels) (N)
a	– acceleration (m/s^2)	R_d	– drivetrain (transmission) resistance (N)
		R_g	– gradient (road slope) resistance (N)

1. Normal load on the front axle

$$W_f = \frac{Wl_2 \cos \theta_s - R_d h_a - haW/g - R_d h_d \pm Wh \sin \theta_s}{L} \text{-----(3)}$$

When the vehicle is climbing up a hill, the **negative sign** (–) used for the term $Wh \sin \theta_s$.

2. Normal load on the rear axle

Similarly, the normal load on the rear axle can be determined by summing moments about point B :

$$W_r = \frac{Wl_1 \cos \theta_s + R_d h_a + haW/g + R_d h_d \pm Wh \sin \theta_s}{L} \text{-----(4)}$$

In the above expression, the **positive sign** is used for the term $Wh \sin \theta_s$, when the vehicle is climbing up a hill.

Where:

W_f	– normal (vertical) load on the front axle, N	g	– acceleration due to gravity, m/s ²
W_r	– normal (vertical) load on the rear axle, N	R_d	– drawbar (hitch) resistance force, N
W	– weight of the vehicle, N	h_d	– height of application of drawbar resistance, m
l_1	– distance from the center of gravity to the rear axle, m	h	– height of the center of gravity, m
l_2	– distance from the center of gravity to the front axle, m	θ_s	– road slope angle
R_a	– aerodynamic resistance (drag force), N	L	– wheelbase of the vehicle, m
h_a	– height of application of aerodynamic resistance, m		
a	– acceleration, m/s ²		

- For small angles of slope, $\cos\theta_s \approx 1$ may be assumed.
- For passenger cars, the height of the point of application of the aerodynamic resistance h_a and that of the drawbar hitch h_d may be assumed to be near the height of the center of gravity h .
- With these simplifications and assumptions, Eqs. (3) and (4) may be rewritten as

**Normal load
on the front axle**

$$W_f = \frac{l_2}{L} W - \frac{h}{L} \left(R_a + \frac{aW}{g} + R_d \pm W \sin \theta_s \right) \quad (5)$$

**Normal load
on the rear axle**

$$W_r = \frac{l_1}{L} W + \frac{h}{L} \left(R_a + \frac{aW}{g} + R_d \pm W \sin \theta_s \right) \quad (6)$$

- By substituting Eq. (2) into the above equations, the following expressions are obtained.

Where:

W_f – normal (vertical) load on the front axle, N
 W_r – normal (vertical) load on the rear axle, N
 W – weight of the vehicle, N
 l_1 – distance from the center of gravity to the rear axle, m
 l_2 – distance from the center of gravity to the front axle, m
 L – wheelbase of the vehicle, m

h – height of the center of gravity, m
 R_a – aerodynamic resistance (drag force), N
 R_d – drawbar (hitch) resistance force, N
 a – acceleration, m/s²
 g – acceleration due to gravity, m/s²
 θ_s – road slope angle

Substituting Eq.2 into Eqs. (5) and (6), the following expressions are obtained:

**Normal load
on the front axle**

$$W_f = \frac{l_2}{L} W - \frac{h}{L} (F - R_r) \quad \text{----- (7)}$$

and

**Normal load
on the rear axle**

$$W_r = \frac{l_1}{L} W + \frac{h}{L} (F - R_r) \quad \text{----- (8)}$$



- It should be noted that the first term on the right-hand side of each equation represents the **static load on the axle** when the vehicle is at rest on level ground.

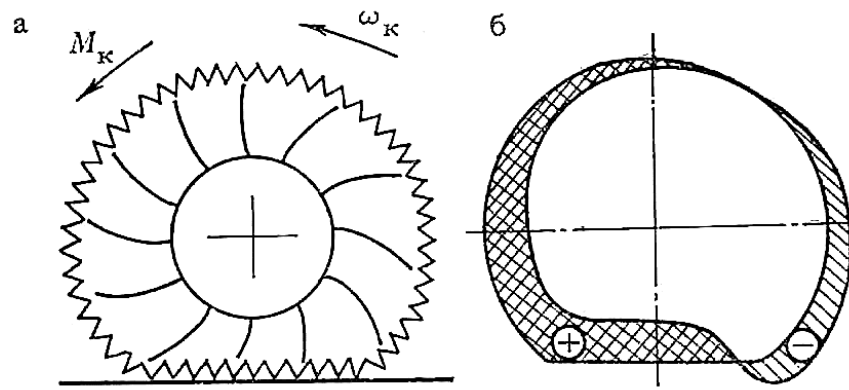


- The second term on the right-hand side of each equation represents the **dynamic component** of the normal load or **dynamic load transfer**.

Where:

W_f – normal (vertical) load on the front axle
 W_r – normal (vertical) load on the rear axle
 W – weight of the vehicle
 l_1 – distance from the center of gravity to the rear axle
 l_2 – distance from the center of gravity to the front axle

L – wheelbase of the vehicle
 h – height of the center of gravity
 F – tractive (driving) force
 R_r – rolling resistance force
(road resistance force)



Tire deformation (a) and diagram (b) under the influence of a turning moment:

+ - compression zone; - - elongation zone

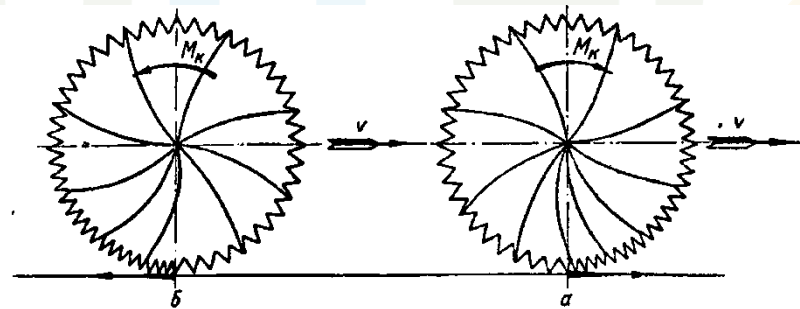


Diagram of the deformation of the wheel surface during traction and braking.

Rolling resistance force

$$P_f = fR_z \cos \alpha = fG_k \cos \alpha$$

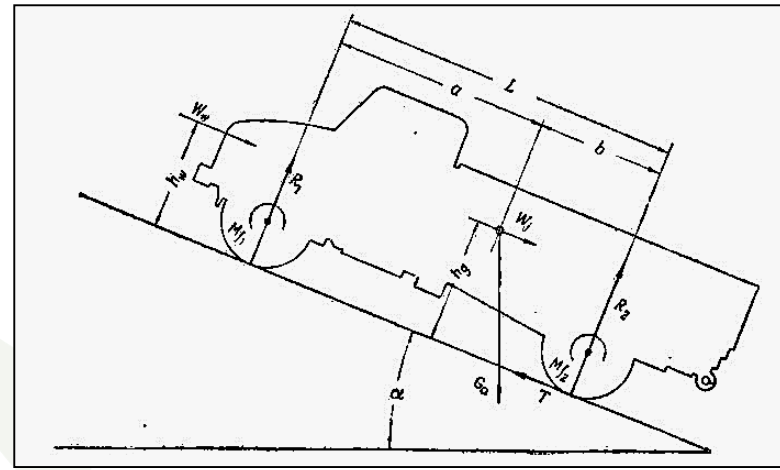
f – Coefficient of Drag Resistance

$R_z \cos \alpha$ – Vertical vertical when pressing the wheel against the base surface - normal force

G_k – Weight that falls on the wheel on the horizontal ground

$\cos \alpha$ – Cosine of the slope angle along the road

Resistance strength of the oblique to automobile behavior



$$P_n = G_a \sin \alpha = G_a i = m_a g \sin \alpha$$

P_n – The force of resistance of the car to climb the slope

G_a – Weight of the car

m_a – vehicle mass

α – Angle of longitudinal slope of the path

i – Value of angle of slope of the road $\tan \alpha$

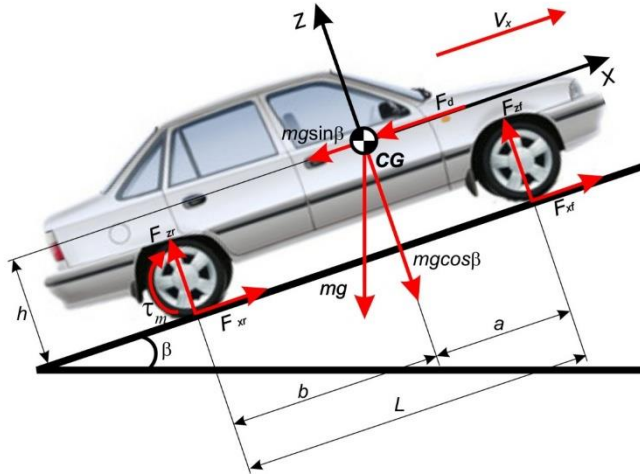
$P_{\psi} = fG_a \cos\alpha + G_a \sin\alpha = G_a(f \cos\alpha + \sin\alpha) = G_a \psi$

$(f \cos\alpha + \sin\alpha)$ – Coefficient of Road Resistance to Traffic

The diagram illustrates a car on an inclined plane. A coordinate system is established with the x-axis along the incline, the z-axis perpendicular to the incline, and the y-axis pointing out of the page. The center of gravity (CG) is marked on the car. Forces acting on the CG are: a normal force F_n perpendicular to the incline, a gravitational force $mg \sin \beta$ acting down the incline, and a friction force F_d acting up the incline. At the rear wheel, a driving force F_{xd} acts up the incline. At the front wheel, a resistance force F_{xl} acts down the incline. The velocity vector V_x is shown pointing up the incline.

$$P_{\psi}=fG_a \cos\alpha + G_a \sin\alpha = G_a(f \cos\alpha + \sin\alpha)= G_a\psi$$

A diagram of a car on an inclined plane at an angle β . The center of gravity (CG) is at a height h from the rear axle. The distance from the rear axle to the CG is b , and the distance from the CG to the front axle is a . The total wheelbase is L . Forces shown include gravity mg acting downwards from the CG, its components $mg \sin \beta$ (parallel to the incline, pointing down) and $mg \cos \beta$ (normal to the incline, pointing down-left). At the rear axle, there is a normal force F_{rr} acting perpendicular to the incline (up-left), a friction force F_{xr} acting parallel to the incline (up-right), and a torque τ_m acting counter-clockwise. At the front axle, there is a normal force F_{rd} acting perpendicular to the incline (up-right), a friction force F_{xd} acting parallel to the incline (up-right), and a drag force F_d acting parallel to the incline (down-right). A velocity vector V_x is shown pointing up the incline. A coordinate system with x and z axes is centered at the CG, with x along the incline and z perpendicular to it.



Thanks for your attention!

